PROBLEMS OF THEORY AND PRACTICAL USE OF A LASER DOPPLER VELOCIMETER IN THE INVESTIGATION OF TURBULENT FLOWS

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At present great attention is being devoted to the development of laser Doppler velocimeters (LDV) for experimental investigation of turbulent flows. The main types of optical schemes of LDV are discussed in this article. Formulas for the output signal of the photoreceiver and the results of spectral analysis of this signal with the statistics of the scattering centers taken into consideration are presented. The uncertainty relation for the spatial resolution and the width of the Doppler spectrum is obtained. The optical scheme of a three-component velocimeter and the block diagram of its electronic part are given. The potentialities of the meter are demonstrated using, as an example, flow past a cylinder.

The operation of an LDV is based on the separation of the Doppler frequency shift in the scattered light, which is proportional to the projection of the velocity of the scattering object on a chosen direction and is given by the formula

$$\omega_D = \mathbf{V}\cdot(\mathbf{K}_s - \mathbf{K}_i) \tag{1}$$

where $\mathbf{V}$ is the velocity vector of the scattering particles in the investigated region of the flow and $\mathbf{K}_i$ and $\mathbf{K}_s$ are wave vectors of the incident and scattered beams, respectively.

The Doppler frequency shift in the scattered light can be extracted by the method of optical heterodyning (homodyning) or with the use of a spectrometer for which a Febry–Perot interferometer can be used. All the different optical schemes of LDV using the heterodyne principle can be grouped in the following main types:

1) schemes with the scattering geometry proposed by Goldstein and Kreid [2] (Fig. 1a);

2) schemes with scattering geometry first described by Yeh and Cummis [1] (Fig. 1b);

3) differential schemes [3, 5] (Fig. 1c, d).

In the schemes of type 1) two coherent beams are directed into the region of the flow being investigated; one of the beams has intensity much smaller than the other and is a reference beam. In the plane of the photoreceiver the reference beam interferes with the scattered beam spatially matched with it. As a mixer of optical signals the photoreceiver separates out the signal of the difference frequency which is equal to the Doppler shift in the scattered beam. An advantage of this scheme is the automatic spatial matching of the beams, whereas in the scheme 2) (Fig. 1b) a special interferometric device is used for this purpose. A characteristic feature of the schemes of both types is the dependence of the parameters of the Doppler signal (frequency, width of the spectrum) on the geometry of both the incident and the scattered beams.

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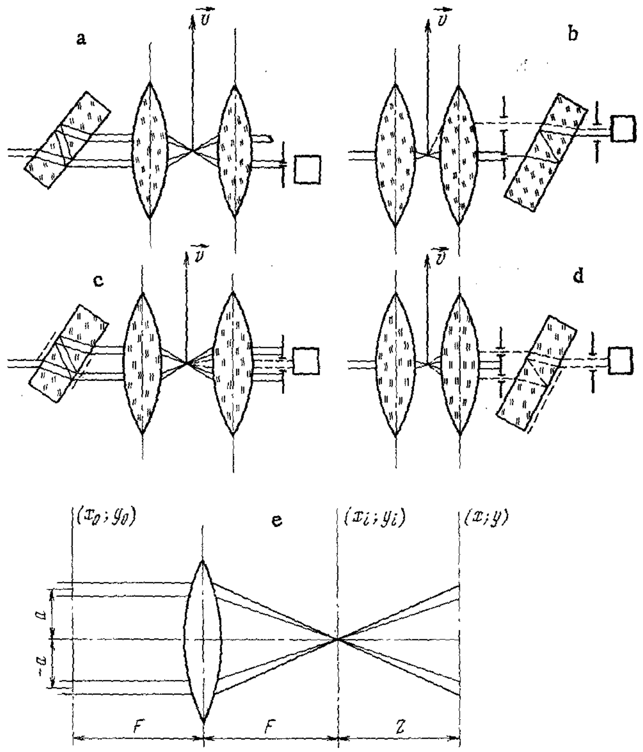


Fig. 1

In the differential scheme (Fig. 1c) two coherent beams of equal intensity are directed into the region of the flow under investigation. The difference Doppler frequency at the output of the photoreceiver does not depend on the geometry of the scattered beams and is determined only by the geometry of the incident beam. The differential scheme shown in Fig. 1d is the inverted scheme of Fig. 1c. In this scheme the difference Doppler frequency and the width of the spectrum do not depend on the geometry of the incident beam and are determined only by the geometry of the scattered beams. The frequency of the signal is equal to the algebraic difference of the Doppler frequency shifts of each scattered beam.

The types of optical schemes of LDV described above can be realized with the division of the amplitude as well as with the division of the wave front of the laser beam.

Analytical Description and Spectral Analysis of Doppler Signal. The structure of the optical signal of an LDV can be analyzed using the tools of the theory of optical filtration. The feasibility of this approach was first pointed out in [6]. A generalized scheme of the device shown in Fig. 1d was chosen for the investigation. Let the field

$$u_{01}(x_0 - a) + u_{02}(x_0 + a)$$

where

$$u_{01}(x_0 - a,\ y_0) = u_{01}(x_0 - a)\, u_{01}(y_0)$$
$$u_{02}(x_0 + a,\ y_0) = u_{02}(x_0 + a)\, u_{02}(y_0)$$

exist in the front Fourier plane (x_0, y_0) of the focussing objective.

We shall regard the scattering center moving in the rear Fourier plane in the direction of the x_i axis as a filter with frequency-space characteristic in the form of delta-function $\delta[x_i - v(t - t_0)]$. If the aperture function of the objective is neglected, the field in the rear Fourier plane can be expressed in the form

$$u(x_i, y_i) = \frac{1}{j\lambda F}\left[\Phi_{01}\exp\left(j\frac{kx_i a}{F}\right) + \Phi_{02}\exp\left(-j\frac{kx_i a}{F}\right)\{1 - \delta[x_i - v(t - t_0)]\}\right] \tag{2}$$

where Φ_{01} and Φ_{02} are Fourier transforms of the functions $U_{01}(x_0, y_0)$ and $U_{02}(x_0, y_0)$ respectively, and F is the focal length of the objective.

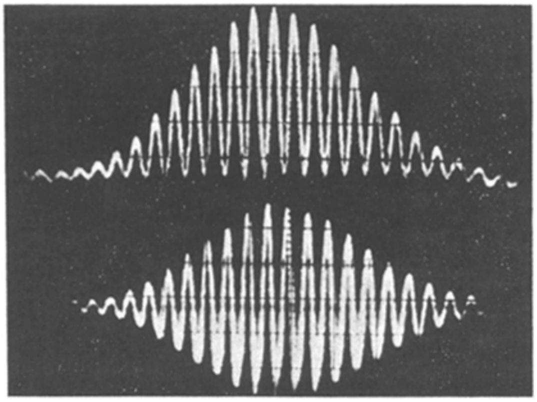


Fig. 2

The field in an arbitrary plane x at a distance z from the focus represents an inverse Fourier transform of field (2) and is written in the following way:

$$U(x,z) = -\frac{\exp(jkz)}{\lambda^2 Fz}\Big\{U_{01}\Big[\Big(x+\frac{az}{F}\Big),y\Big] + U_{02}\Big[\Big(x-\frac{az}{F}\Big),y\Big] - U_{01}(y)\,\Phi_{01}\Big[\frac{kv}{F}(t-t_0)\Big]\exp\Big(j\frac{kv}{F}\Big)(t-t_0)(a+x) - U_{02}(y)\,\Phi_{02}\Big[\frac{kv}{F}(t-t_0)\Big]\exp\Big(-j\frac{kv}{F}\Big)(t-t_0)(a-x)\Big\} \tag{3}$$

In the differential scheme the photoreceiver located at a point x (outside the direct transmitted rays) records the intensity of the field diffracted at the scattering center. The expression for it (its intensity) is easily obtained from (3):

$$I_g = \frac{1}{\lambda^4 Fz}\Big\{U_{01}(y)\,\Phi_{01}^2\Big[\frac{k}{F}v(t-t_0)\Big] + U_{02}(y)\,\Phi_{02}^2\Big[\frac{k}{F}v(t-t_0)\Big] + 2U_{01}(y)\,U_{02}(y)\,\Phi_{01}\Big[\frac{k}{F}v(t-t_0)\Big]\cos[\omega_D(t-t_0)]\Big\}$$
$$\Big(\omega_D = \frac{2akv}{F}\Big) \tag{4}$$

It follows from (4) that the Doppler frequency does not depend on the coordinate x. The field intensity recorded by the photoreceiver at a point $x = az/F$ in the scheme with the reference beam is given by an expression obtained from (3) under the condition $U_{01} \gg U_{02}$:

$$I_{0n} = \frac{1}{\lambda^4 Fz}\Big\{U_{02}^2\Big[\Big(x-\frac{az}{F}\Big),y\Big] - U_{01}^2(y)\,\Phi_{01}^2\Big[\frac{kv}{F}(t-t_0)\Big] - 2U_{02}\Big[\Big(x-\frac{az}{F}\Big),y\Big]U_{01}(y)\,\Phi_{01}\Big[\frac{kv}{F}(t-t_0)\Big]\cos[\omega_D(t-t_0)]\Big\}$$
$$\Big(\omega_D = \frac{kv}{F}(a+x)\Big) \tag{5}$$

It is evident from (5) that the value of the Doppler frequency in the scheme with the reference beam depends on the coordinate x and, hence, on the geometry of the scattered beam.

From (4), (5) we find that the condition of best contrast in the differential scheme is

$$U_{01}(y)\,\Phi_{01} = U_{02}(y)\Phi_{02} \tag{6}$$

while in the scheme with the reference beam it is

$$U_{02}[(x-a),\ y] = U_{01}(y)\,\Phi_{01} \tag{7}$$

In real optical schemes of an LDV at the input there are either two gaussian beams (schemes with division of the field amplitude) or two beams diffracted at identical slits (diaphragms). Accordingly, the expression for $\Phi[kvF^{-1}(t-t_0)]$ in (3), (4), and (5) has the form

for gaussian beams

$$\Phi(t) = \sqrt{\frac{\pi}{2}}\exp[-\xi^2\omega_D^2(t-t_0)^2] \qquad \Big(\xi = \frac{\sigma}{2\sqrt{2a}}\Big) \tag{8}$$

for two identical circular diaphragms of diameter b with uniform distribution of the intensity

$$\Phi(t) = 4a\frac{J_1[b\omega_D a^{-1}(t-t_0)]}{\omega_D(t-t_0)} \tag{9}$$

for two identical slits of width b with uniform distribution of the intensity

$$\Phi(t) = 4a\frac{\sin[b\omega_D a^{-1}(t-t_0)]}{\omega_D(t-t_0)} \tag{10}$$

where σ is the parameter of the intensity distribution of the input gaussian beam; J_1 is a Bessel function of first kind and first order.

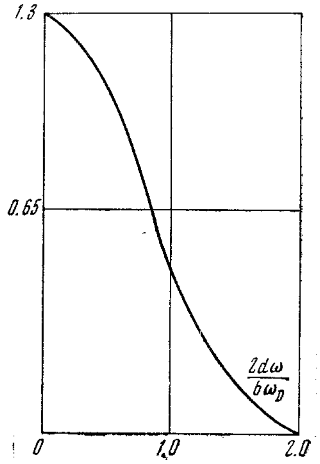


Fig. 3

The signal from a single particle, obtained experimentally in the scheme with two slits before and after subtracting the constant component, is shown in Fig. 2.

As follows from formulas (4)-(7) under the condition of maximum contrast of the interference field in the plane of the photoreceiver, the total Doppler signal can be written in the form

$$I(t) = \sum_{n=1}^{N} I_n(t - t_n)\{1 + \cos[\omega_D(t - t_n)]\} \tag{11}$$

where t_n is the random instant of appearance of the n-th scattering particle in the region of intersection of the incident beams in the flow and N is the number of particles that have passed through the investigated volume. The signal consists of the intrinsic Doppler component

$$\sum_{n=1}^{N} I_n(t - t_n)\cos(\omega_D(t - t_n)]$$

and a low-frequency "constant" component

$$\sum_{n=1}^{N} I_n(t - t_n)$$

As shown below, under the condition that the scattering point particles form a uniform Poisson field in the investigated flow, the spectrum of the signal is described by the expression

$$I(\omega) = \frac{1}{T}|I_0(\omega)|^2\left[Tq + 2\frac{q^2}{\omega^2} - 2e^{-\alpha}\left(\frac{q^2}{\omega^2}\cos\beta + \frac{q}{\omega}\sin\beta\right)\right]$$
$$\left(\alpha = \frac{\omega^2 Tq}{q^2 + \omega^2}, \qquad \beta = \frac{\omega Tq^2}{q^2 + \omega^2}\right) \tag{12}$$

where q is the flux density of the particles, T is the period of analysis of the signal, and $I_0(\omega)$ is the amplitude spectrum of the signal from a single particle.

From (8)-(10) we easily find that for gaussian beams

$$I_0(\omega) = \sum_{i=1}^{2} \exp[\omega_i^2/(4\xi\omega_D^2)] \qquad (\omega_1 = \omega,\ \omega_2 = \omega - \omega_D = \Delta\omega) \tag{13}$$

For schemes with circular diaphragms

$$I_0(\omega) = \sum_{i=1}^{2} \xi_i(\omega_i)\left[\arccos\left|\frac{a\omega_i}{2b\omega_D}\right| - \frac{a\omega_i}{2b\omega_D}\sqrt{1 - \frac{b^2\omega_i^2}{4b^2\omega_D^2}}\right]$$
$$\xi_i(\omega) = \begin{cases} 1 & \text{for } |\omega_i| \leqslant 2b\omega_D/a \\ 0 & \text{for } |\omega_i| > 2b\omega_D/a \end{cases} \tag{14}$$

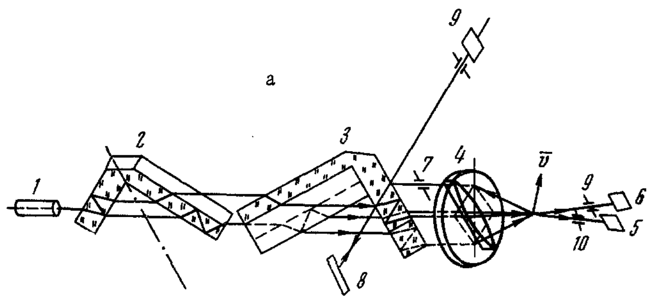
For schemes with slits

$$I_0(\omega) = \sum_{i=1}^{2} \xi_i(\omega_i)\left[\frac{a}{b}\omega_D - |\omega_i|\right] \tag{15}$$

The expression for the spectrum of the Doppler signal with the constant component filtered out becomes highly simplified:

$$I(\Delta\omega) = qI_0^2(\Delta\omega) \tag{16}$$

It follows from (16) that the energy spectrum of the signal repeats the energy spectrum of the signal from a single particle and is proportional to the concentration of the particles. Hence the root-mean-square value of the variable part or the Doppler signal is proportional to $\sqrt{q}$. The amplitude spectrum computed from formulas (14) and (16) is shown in Fig. 3 by way of an example.



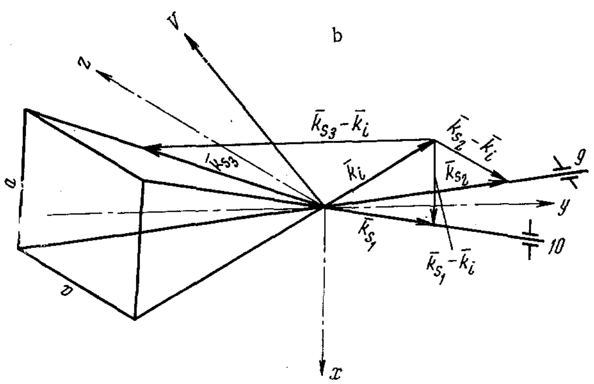


Fig. 4

Relation between Spatial and Temporal Resolution of an LDV. Analysis of formulas (8)-(10) and (13)-(15) shows that the relation between the spatial resolution of an optical scheme of the meter and the width of the amplitude spectrum can, in general, be written in the form

$$\delta\Delta f = \beta v \tag{17}$$

where δ is the minimum linear dimension from which information about the velocity is extracted, Δf is the halfwidth of the amplitude spectrum, v is the velocity, and β is a constant coefficient which depends on the type of the optical scheme and the level to which the width of the spectrum is referred.

For example, for schemes with circular diaphragms $\beta = 7.6/\pi$, while for schemes with rectangular slits $\beta = 2$. The spatial resolution is estimated from the zeros of the first maximum of the Airy function, while the width of the spectrum is estimated from the zero level of the spectral density; for gaussian beams with δ and Δf reckoned from the l^{-2} level, $\beta = 2/\pi$.

During the use of an LDV for the investigation of turbulent flows the instantaneous frequency of the Doppler signal changes in proportion to the fluctuations of the velocity, i.e., the signal becomes frequency modulated. It is known that the spectrum of a frequency-modulated signal is a complex function of the modulating process which is simplified only in two cases: if the modulation is narrowband, i.e., if the intensity of the turbulence is small and the modulating process is a high-frequency one, then one-half of the Doppler spectrum repeats the spectrum of the modulating process; if the turbulence is strong and the process is a low-frequency one, then the Doppler spectrum repeats the curve of distribution of the velocity values. In the general case, it is difficult to infer the parameters of the modulating process from the nature of the spectrum of the frequency-modulated oscillation. In view of this, the use of a spectrum analyzer as a measuring device in Doppler systems of investigation of turbulence is not promising.

Three-Component LDV. Optical Part. On the basis of the investigations carried out by the authors a new scheme of LDV is proposed which permits one to measure the three orthogonal projections of the velocity vector, and the principles of a rational analysis of the obtained Doppler signal have been worked out. A description of the prototype of the equipment is given below.

The optical part is shown in Fig. 4. The ray from laser 1 is divided by two splitters 2 and 3 placed in succession into three parallel beams located in pairs in the orthogonal planes. The intensity of the beam, directed along the sides of the right angle formed by these orthogonal planes, is much greater than

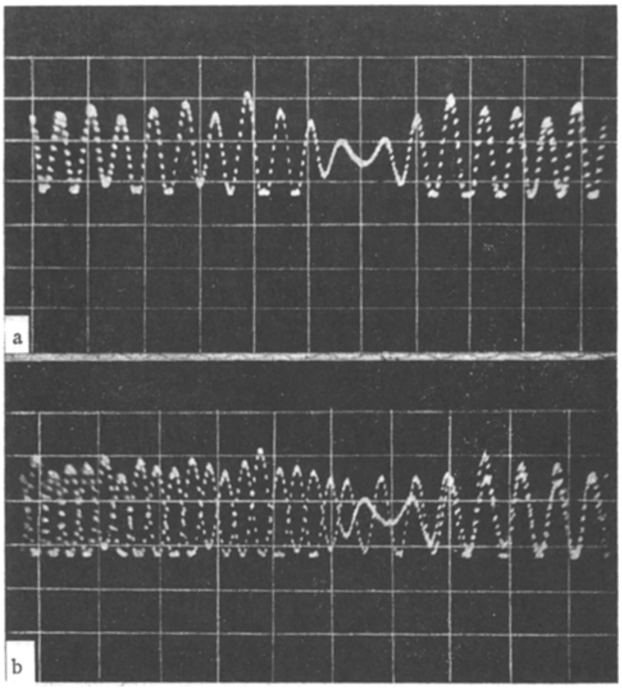


Fig. 5

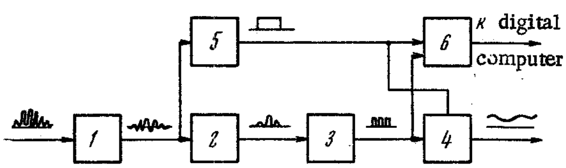


Fig. 6

the intensities of the other two beams which play the role of reference beams. The split beams are focussed at the investigated region of the flow by an objective 4. The light beams scattered by the impurity particles in the focal plane of the objective have a Doppler frequency shift which is proportional to the velocity of the scatterers. The extraction of the Doppler shift is done by optical heterodyning of spatially matched reference and scattered beams by the photoreceivers 5 and 6. The frequency of the obtained signals is proportional to the projections of the velocity vector on the corresponding difference vectors $\mathbf{K}_{S1} - \mathbf{K}_i$ and $\mathbf{K}_{S2} - \mathbf{K}_i$.

In order to determine the third projection, a scattered beam, separated out by diaphragm 7 in the direction symmetric to the direction of the strong incident beam with respect to the axis of the objective and lying in the same plane, is used. The Doppler shift is equal to the projection of the velocity vector $\mathbf{K}_{S3} - \mathbf{K}_i$ and is separated out by heterodyning in a Michelson interferometer. It is easy to see from Fig. 4 that these difference vectors are mutually orthogonal. Thus, this optical scheme of an LDV permits a simultaneous measurement of the three orthogonal projections of the velocity vector.

A typical signal obtained from the photoreceiver in the case of a laminar flow is shown in Fig. 5. As expected, it has the nature of beats strongly modulated in amplitude and has a parasitic phase modulation caused by the random nature of the instant of entrance of the particles in the focal plane of the objective. The nature of the phase modulation is clearly seen in Fig. 5b, where a superposition of the halves of one and the same realization of the signal is shown.

Electronic Part. The first variant of the equipment was constructed based on a circuit with phase automatic frequency control described in [8]. This circuit was found to be sensitive to sharp oscillations of the signal amplitude and phase jumps. As a result, for a laminar flow the noise in the 50-Hz band was 3% in scaling to equivalent turbulence. In view of the large value of noises, it became necessary to introduce into the system elements of nonlinear filtration and of tracking frequency and not phase.

The block diagram of the second variant of the meter is shown in Fig. 6. The signal from the photoreceiver arrives at a controlled band-pass filter 1 for the suppression of the noises of the laser and the photoreceiver. Then it is fed to a threshold amplifier 2 which passes signal only in those time segments when its amplitude far exceeds the noise level. A pulse shaper 3 connected after the amplifier transforms the signal into a pulse train. Simultaneously, a pulse shaper 5 produces strobe pulses with duration

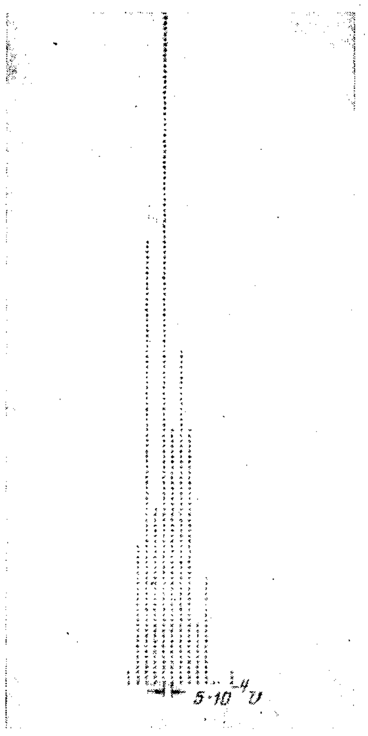


Fig. 7

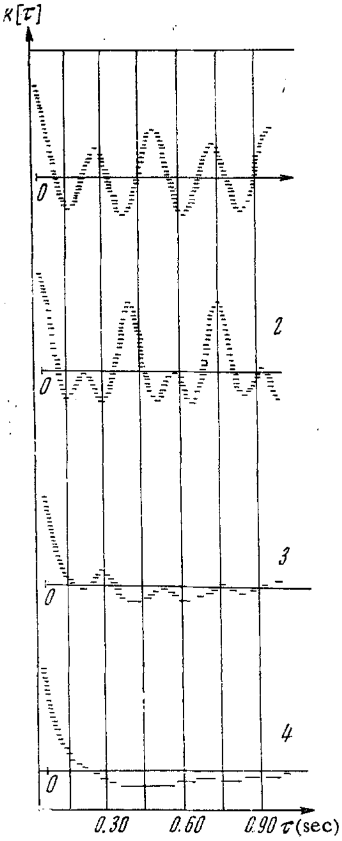


Fig. 8

equal to the duration of the train. The pulse trains arrive at a frequency detector 4 with memory, which operates at the instant of disappearance of the signal. An analog signal proportional to the instantaneous value of the velocity of the investigated flow is obtained at the output of the frequency detector. The pulse trains are fed also to a counter 6 controlled by strobe pulses in such a way that the prespecified counting period is made longer in the time segments, in which there is no signal at the output of the counter. As a result a code of numbers equal to the value of the mean velocity for the chosen period is obtained at the output of this device. The counter is equipped with a digital signal panel showing the instantaneous values of the velocity. The code voltages from the output of the counter are fed to a computer which gives out all the statistical characteristics of the turbulent flow.

As an example, the distribution function of the values of the velocity for an averaging time of 1 sec obtained with a digital computer for a laminar flow with a velocity of 1 cm/sec is shown in Fig. 7. The dispersion in this case is caused by the equipment noise. The root-mean-square deviation is of the order of 0.1%.

The equipment with a single channel of measurement has the following characteristics:

1) the range of measurable values of velocity is from $2 \cdot 10^{-3}$ to 2 m/sec;

2) the equipment noise, equivalent to turbulence in the frequency band 0-100 Hz, is 1%;

3) the pass band for the frequency of fluctuations of the velocity is divided into three subranges of 10, 100, and 1000 Hz;

4) the spatial resolution is of the order of $0.1 \times 0.1 \times 0.5$ mm^3;

5) the averaging time is 10^{-3}–10 sec.

Experimental Results. The following experiment was conducted in order to illustrate the potentialities of a single measuring channel. In a channel with cross section 16×16 mm with laminar flow a cylinder of 5 mm diameter was placed at 40 mm upstream from the focus of the LDV along the vertical axis of the channel. The analog signal from the output of the LDV was fed to a correlator. The autocorrelation functions of Karman vortex street are shown in Fig. 8 for average velocities of: 1) 3.5 cm/sec; 2) 5 cm/sec; 3) 7 cm/sec; and 4) 10 cm/sec. For the lowest velocity the size of the vortex determined from the formula R = VT, where V is the average velocity measured by LDV and T is the period of the correlation function, is equal to 7 mm.

With the increase of the velocity the size of the vortex decreases and finally the correlation function loses its periodicity, i.e., the regular process is disrupted.

In conclusion, we consider some additional problems connected with the method discussed above.

Signal Spectrum from a Single Particle. In order to determine the amplitude spectrum of the Doppler signal from a single particle in the case of cylindrically symmetric beams it should be taken into consideration that the spectrum of the function $[J_1(\rho t)]/t$ is

$$S(\omega) = \frac{\sqrt{\pi}\, 0.886}{2\rho} \sqrt{\rho^2 - \omega^2} \tag{18}$$

Therefore, the function $[J_1^2(\rho t)]/t^2$ corresponds as a Fourier transform to the convolution

$$G(\omega) = \int_{-\infty}^{\infty} S(\gamma) S(\gamma - \omega)\, d\gamma \tag{19}$$

Geometrically this convolution represents the area of a region common to two circles, from which it follows (for $\omega < 2\rho$) that:

$$G(\omega) = \left(2 \arccos \frac{\omega}{2\rho} - \frac{\omega}{\rho} \sqrt{1 - \frac{\omega^2}{4\rho^2}}\right) a^2 \tag{20}$$

The expression for $I_0(\omega)$ in the case of rectangular slits is obtained in a similar way. Here it is taken into consideration that the function $[2 \sin(\rho t)]/\pi t$ is the inverse Fourier transform of a solitary rectangular pulse of length ρ in frequency space. Hence the Fourier transform of the function $[4 \sin^2(\rho t)]/\pi^2 t^2$ is the convolution of the rectangular pulse with itself, which gives the triangle

$$G(\omega) = 2\rho - |\omega| \tag{21}$$

Signal Spectrum with the Statistics of the Particles Taken into Consideration. If the flux of the scattering particles is assumed to be a uniform Poisson flux, the time lag between the instants of falling of l-th and $l + 1$-st particles into the operating zone will be a continuous random quantity distributed according to the power law

$$f(t_l) = \begin{cases} 0 & \text{for } t_l < 0 \\ q e^{-q t_l} & \text{for } t_l > 0 \end{cases}$$

Since the instant of falling of the n-th particle in the interference region is $t_n = \sum_{l=1}^{n} t_l$, the density of distribution of the random quantity t_n is determined according to the Erlang law

$$f(t_n) = \frac{q (q t_n)^{n-1}}{(n-1)!} e^{-q t_n} \tag{22}$$

The number of particles N during the period of observation T will be a random quantity distributed according to Poisson's law with the density

$$f(N) = \frac{(Tq)^n}{N!} e^{-Tq} \tag{23}$$

Using (23), the theorem of lag, and the theorem of the spectrum of a sum, we can write the expression for the amplitude spectrum of the Doppler signal in the following way:

$$I(\omega) = I_0(\omega)\left(1 + \sum_{n=1}^{N} e^{-j\omega t_n}\right) \tag{24}$$

where $I_0(\omega)$ is the spectrum of the signal from a single particle and t_n is the instant of appearance of the n-th particle at the center of the region of information collection.

Averaging (24) over t_n and N we easily obtain the mathematical expectation of the spectrum

$$\langle I(\omega)\rangle = \frac{1}{T} I_0^2(\omega)\left[Tq + 2\frac{q^2}{\omega^2} - 2e^{-\alpha}\left(\frac{q^2}{\omega^2}\cos\beta + \frac{q}{\omega}\sin\beta\right)\right]$$

$$\left(\alpha = \frac{\omega^2 Tq}{q^2+\omega^2}, \quad \beta = \frac{\omega T q^2}{q^2+\omega^2}\right) \tag{25}$$

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